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Spacecraft In-Orbit Identification Using Eigensystem Realization Methods

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In this paper two multi-input/multi-output state space methods suitable for the in-orbit system identification of space structures are considered. The ERA (eigensystem realization algorithm) method, which has already been used successfully on the Solar Array Flight Experiment, is compared with the ERA/DC (ERA using data correlations) method in terms of the formulation and computational effort. The methods are applied to simulated data from a space station type model. The ERA/DC method requires less model overspecification in the presence of noise than does the ERA method for similar quality of results. A revision of the ERA/DC method is suggested.

Introduction

FUTURE large space structures, such as the Space Station, will require in-orbit system identification of their structural dynamic behavior to validate theoretical models and to insure that control systems do not give rise to any instabilities associated with control-structure interaction.[1] This validation is particularly important as structures become larger and more flexible since the control bandwidth can approach the first structural frequency. Typically, an identified model for control synthesis should represent the modal behavior of the structure in a frequency range up to 10 times the control bandwidth.

The need for in-orbit testing arises for two reasons. First, the structures will be so complex that theoretical finite element type models will not be sufficiently accurate to be relied on in the absence of test data; in particular the modeling of joints between substructures and of damping is extremely difficult. Second, even substructures will usually be so large and flexible that they will require multiple supports if they are to be tested in a 1*g* environment. Therefore, in most cases ground-based modal test results for substructures will be unlikely to predict in-orbit behavior with sufficient accuracy, given also the uncertainty in the stiffness and damping of the joints between substructures. The use of scale models, while valuable in gaining understanding of the behavior of space structures, will not obviate the need for in-orbit identification.

The in-orbit identification of future structures like the Space Station will prove extremely challenging due to the large number of low-frequency, lightly damped, and very closely spaced modes of vibration. Any test will not only require multiple response measurements but also multiple excitation force locations and multiple sets of inputs (or references) to allow identification of the very close modes. Thus multi-input/multi-output (MIMO) identification methods are required. The use of multi-point excitation is well established in the aerospace industry for ground vibration testing of aircraft. However, the use of such steady state sinusoidal excitation techniques in space is considered impractical because test times will be excessive and the energy expended may be prohibitive. Instead the application of various patterns of multiple simultaneous impulses would seem more appropriate, the sequence of patterns designed to minimize the net rigid body response of the space structure. The test philosophy would depend upon whether the modes of a component (such as the Solar Array deployed from the Space Shuttle) or of an entire free-free structure were required.

The mathematical model to be identified will be nominally linear provided that rigid body rotational motion is small. However, experience on the Solar Array Flight Experiment (SAFE)[2] indicated significant variation of dynamic characteristics with response amplitude, and it was said in Ref. 3 that the polyreference method could not be applied to any combination of excitations because frequencies and dampings varied significantly from test to test (this difficulty would have been found with any of the linear identification methods). This variation means that test inputs will need to be designed carefully in order to minimize nonlinear effects. Possibly tests will need to be carried out at several excitation amplitudes in order to investigate the nonlinear characteristics.

The most common mathematical model proposed for identification is the linear state space representation based on a set of first-order differential equations. This representation has the advantage that only one type of data (e.g., acceleration) needs to be used. The size of the model is twice the number of modes required and the outputs and inputs could be only those required for control purposes. Transient data may also be used.

The lumped parameter model, however, is represented by a set of second-order differential equations whose coordinates are responses at various positions on the structure; the mass, stiffness, and damping matrices corresponding to these positions are then identified directly.[4] Both time and frequency domain methods have been proposed. The number of measurement positions must be at least equal to the number of modes of interest. Also, the displacement, velocity, and acceleration at each position need to be measured or else estimated by time or frequency domain integration, which is not simple for free decay data. The main use of such models is to update the theoretical finite element representation; it has been suggested that a state space method be utilized to provide the measure modal results required for such a modification.[5]

A further form of mathematical model has been proposed, and this is the distributed parameter model based on the partial differential equations of the components; only the coefficients of the equations are then identified.[6] Such a model type seems promising for space structures that often have components of simple geometry and essentially uniform section, such as trusses and solar arrays.

However, in this paper some methods that are based on the state space equation will be considered, in particular those methods based in the time domain. Several such methods have been applied to data obtained in orbit, namely the least squares complex exponential, polyreference, Ibrahim time domain and eigensystem realization algorithm (ERA) meth-

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ods.[1-3] All of these methods are essentially based on the least squares fit[7,8] to a discrete state space model, originally proposed in the control field. However, the ERA method[9] is most easily recognizable as a state space method in its formulation. A major disadvantage when using all of these methods is that biased parameter estimates are found when noise is present and the true model order is used. There are various approaches in the system identification literature for reducing bias.[10] In the aforementioned least squares based methods, the bias is reduced by overspecification of the model order used in the solution (similar to the repeated least squares method), but this approach means that additional "spurious modes" are generated and need to be distinguished from the genuine modes. This task is not straightforward, and various confidence factors are used to assist in the process. The ERA method is unique among the least squares methods in that, after initial model overspecification, a singular value decomposition (SVD) process is incorporated in the algorithm to reduce the model to a "minimum" order and so prevent the generation of spurious results if the correct final model order is selected.

Another approach to bias reduction is to introduce a noise model and to use a maximum likelihood or other method to identify parameters of the system and noise models simultaneously. Theoretically, this is the optimum approach to identification, but it suffers from various disadvantages for high-order systems with lightly damped close modes. In particular the form and order of the noise model have to be chosen, the number of unknowns is increased significantly by the noise model, and the method is iterative, very computationally intensive, and may not converge unless initial parameter estimates are very close to the final values.[6] However, it is possible that such an approach could be used to "fine tune" the results provided by a simpler and faster method, especially since information on the variance of parameter estimates is given when using the maximum likelihood techniques.

A further approach to reducing bias is to minimize or eliminate the effect of the terms that actually cause the bias, using methods such as instrumental variables or correlation fit.[10] The latter method involves a noniterative least squares curve fit to data correlations and in principle does not require model overspecification. Estimates have been shown to be less sensitive to error for close modes when using correlation fit than when using the classical least squares approach. The ERA and correlation fit philosophies have been combined to yield the ERA using data correlations (ERA/DC) method, a minimum-order state space realization using correlation data instead of time data.[11] There are no approximations involved in the method, and exact parameter estimates for noise-free multimode multi-input/multi-output (MIMO) data are found when using it. The method has been applied to aircraft ground vibration and flight flutter test data[12] and found to produce results as good as, if not better than, the ERA method used by NASA for ground test and for in-orbit identification.

Whereas the ERA/DC curve fits any number of correlations not expected to be corrupted by noise, the q-Markov COVER[13] technique performs a collocation fit to both impulse response and autocorrelations so that the number of points fitted depends on the model size and does not ignore those correlation values likely to be corrupted by noise.

Because the ERA/DC method involves the calculation of correlations before the SVD process, it has been thought by the authors that the method would be significantly more computationally intensive than the ERA method that carries out an SVD on a matrix of impulse response data. Indeed, in Ref. 14, Pinson summarizes the features of the various ERA type methods and categorizes the basic ERA method as providing rapid analysis whereas the ERA/DC method does not. However, the ERA/DC method is indicated as being suitable for small computers; this is because less model overspecification is required than for the ERA method and so smaller matrices may be employed. However, what the authors have found in the computer implementation of the ERA/DC method is that it can in fact be significantly faster than the ERA method.[12]

In this paper the ERA and ERA/DC methods will be considered and applied to simulated multimode MIMO data from a simplified space station type model. The relative quality of results and computation times will be discussed.

Identification Methods

Both techniques considered in this paper produce a consistent set of modal parameters for impulse responses from multiple input cases, measured at multiple response stations. If the ith response at time instant $(k + 1)$ due to the jth input case is $y_{i,j}$, then all of the responses may be written[7] as

$$Y_{k+1} = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,NI} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,NI} \\ \vdots & \vdots & & \vdots \\ y_{NO,1} & y_{NO,2} & \cdots & y_{NO,NI} \end{bmatrix} = CA^kB \tag{1}$$

where Y is an $(NO*NI)$ data matrix, A is the $(M*M)$ system matrix, and C and B are $(NO*M)$ measurement and $(M*NI)$ input matrices, respectively. The M is the system order (twice the number of modes), and NO and NI are the number of measurement stations and input cases, respectively. Given the data sequence $\{Y_k\}$ for a range of sampling instants, a realization (i.e., a solution) can be obtained for the A, B, and C matrices.

By calculating the eigendecomposition of the system matrix A such that

$$A = \psi Z \psi^{-1} \tag{2}$$

then Eq. (1) becomes

$$Y_{k+1} = C\psi Z^k \psi^{-1} B \tag{3}$$

The system frequencies and dampings are found from the eigenvalues of matrix A, which occur in complex conjugate pairs in diagonal matrix Z, via the relationship

$$\lambda_i = \exp(-\zeta_i\omega_i\Delta t \pm \omega_i\sqrt{1-\zeta_i^2}\,\Delta t) \tag{4}$$

where for the ith mode λ is the eigenvalue, ζ the critical damping ratio, ω the natural frequency, and Δt the sampling interval.

The matrix $C\psi$ gives the system mode shapes, and the matrix $\psi^{-1}B$ contains the modal participation factors that indicate how effective a particular input is at exciting each mode.

The formulations of the two identification techniques are now discussed.

Eigensystem Realization Algorithm[9]

A block data Hankel matrix H is constructed using the impulse data sequences such that for the kth time increment

$$H_k = \begin{bmatrix} Y_{k+1} & Y_{k+2} & \cdots & Y_{k+\eta+1} \\ Y_{k+2} & Y_{k+3} & \cdots & Y_{k+\eta+2} \\ \vdots & \vdots & & \vdots \\ Y_{k+\xi+1} & Y_{k+\xi+2} & & Y_{k+\eta+\xi+1} \end{bmatrix}$$

$$= \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\xi} \end{bmatrix} A^k \quad [B \;\; AB \;\; \cdots \;\; A^{\eta}B] = VA^kW \tag{5}$$

where V is a $(\xi + 1)NO*M$ observability matrix, W is an $M*(\eta + 1)NI$ controllability matrix, and ξ and η dictate the dimensions of the $(\xi+1)NO*(\eta+1)NI$ block matrix H. There are other more complicated ways of constructing this matrix that will not be considered here.[9]

The SVD of matrix H_o is found in the form

$$H_o = P_{FH}D_{FH}Q_{FH}^T = P_H D_H Q_H^T \tag{6}$$

where P_{FH} and Q_{FH} are orthogonal matrices of left and right singular vectors, and D_{FH} is a diagonal matrix of singular values. The size of the matrices can be truncated by ignoring those singular values that are considered to be zero, or very small, and omitting the corresponding singular vectors. The truncated matrices are denoted as P_H, D_H, and Q_H. The pseudo-inverse of H_o, denoted as $H^\#$, is defined as

$$H_o H^\# H_o = H_o \tag{7}$$

and can be shown to be

$$H^\# = Q_H D_H^{-1} P_H^T \tag{8}$$

It can be seen that

$$Y_{k+1} = E_{NO}^T H_k E_{NI} \tag{9}$$

where

$$E_{NO}^T = [I_{NO}, O_{NO}, \ldots, O_{NO}] \tag{10}$$

and I_{NO} and O_{NO} are $(NO*NO)$ identity and null matrices, respectively.

Combining Eqs. (5-9) can be shown, after some manipulation, to give the minimum realization

$$Y_{k+1} = E_{NO}^T P_H D_H^{½} [D_H^{-½} P_H^T H_1 Q_H D_H^{-½}]^k D_H^{½} Q_H^T E_{NI} \tag{11}$$

and comparison with Eq. (1) gives realizations of the A, B, and C matrices that can then be used to find the modal parameters as described earlier. For example,

$$C = E_{NO}^T P_H D_H^{½} \tag{12}$$

Although not obvious from its formulation, the ERA method has the least squares technique as its fundamental element[8] (as with other popular modal analysis methods such as polyreference and least squares complex exponential). The computational model therefore needs to be overspecified to reduce the inherent bias on the parameter estimates in the presence of noise, but spurious modal results are introduced. The use of the singular value decomposition theoretically enables all of the spurious modes to be eliminated by truncating the solution order. However, in practice it is impossible to use the singular values to separate all of the system and spurious modes,[15] and so the remaining spurious modes have to be eliminated using other techniques such as modal amplitude coherence.[9]

Eigensystem Realization Algorithm Using Data Correlations[11]

The ERA/DC technique combines the ERA method described earlier with the correlation fit approach that considers the curve fit in terms of the correlations that are used. By using certain correlation values, the ERA/DC technique theoretically does away with the need to overspecify the mathematical model order because the terms that introduce bias in the least squares based methods can be eliminated.

Starting with the block data matrix defined in Eq. (5), a square block correlation matrix R_k of size $\gamma = (\xi+1)*NO$ is defined for the kth time instant as

$$R_k = H_k H_o^T = VA^k WW^T V^T = VA^k W_c \tag{13}$$

and a block correlation Hankel matrix U_q is constructed such that

$$U_q = \begin{bmatrix} R_q & R_{q+r} & \cdots & R_{q+\beta r} \\ R_{q+r} & R_{q+2r} & \cdots & R_{q+(\beta+1)r} \\ \vdots & \vdots & & \vdots \\ R_{q+\alpha r} & R_{q+(\alpha+1)r} & & R_{q+(\alpha+\beta)r} \end{bmatrix} \tag{14}$$

where α and β determine the size of the $(\alpha+1)\gamma*(\beta+1)\gamma$ block correlation matrix. The parameter q is set so that the low lag autocorrelations (where theoretically most of the bias errors are likely to originate) can be omitted, and the r parameter allows the block correlation Hankel matrix to be set up without any overlap between any of the blocks.

The SVD of the U_q matrix is constructed as

$$U_q = P_{Fu}D_{Fu}Q_{Fu}^T = P_u D_u Q_u^T \tag{15}$$

where P_u, D_u, and Q_u are truncated matrices. Recognizing that the pseudo-inverse of U_q *is*

$$U^\# = Q_u D_u^{-1} P_u^T \tag{16}$$

then the expression

$$R_{q+j} = E_\gamma^T U_{q+j} E_\gamma \tag{17}$$

can be expanded in a similar way to the ERA method to give

$$R_{q+j} = E_\gamma^T P_u D_u^{½} [D_u^{-½} P_u^T U_{q+1} Q_u D_u^{-½}]^j D_u^{½} Q_u^T E_\gamma \tag{18}$$

$$= \quad V \qquad\qquad A^j \qquad\qquad A^q W_c$$

Comparing Eqs. (18) and (13) with $k = q + j$ gives realizations of A, V, and $A^q W_c$. The first NO rows of V are the C matrix. Equation (5) can be written as

$$H_o = VW \tag{19}$$

and so it follows that

$$W = (V^T V)^{-1} V^T H_o \tag{20}$$

and thus W can be found using the realization of V. The first NI columns of the W matrix define the B matrix.

Once matrices A, B, and C have been found, the modal parameters are calculated as they are for the ERA technique.

Implementation

ERA Method

The degree of initial model order overspecification is dictated by the smaller dimension of the block data matrix H, in this work the row dimension; it must be at least equal to the system order M but may need to be increased to nearer $3M$ when noise is present.

The greater dimension of H, here the number of columns $(\eta+1)NI$, is an indication of the number of data points from the decay used in the curve fit. The choice of parameters must be such that

$$\zeta + \eta + 3 \le NP$$

where NP is the maximum number of data samples from each response to be included in the analysis; this value should include the significant part of the decay.

The way in which the blocks of the H matrix are set up in this paper is such that the subscript of Y (i.e., the sample number) is incremented equally in both row and column directions. So, for example, with $\xi = 2$ and $\eta = 4$, the samples used in the H_o matrix would be

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \end{matrix}$$

It would also be possible to increment the subscript differently in the row and column directions as in Ref. 9 so that the preceding example could become

$$\begin{matrix} 1 & 4 & 7 & 10 & 13 \\ 2 & 5 & 8 & 11 & 14 \\ 3 & 6 & 9 & 12 & 15 \end{matrix}$$

and more samples could be included for the same size of matrix or possibly a smaller matrix could be used. This block layout needs further consideration. Indeed, NASA apparently goes further and uses two block sizes, one with all outputs and the other with a subset of outputs. Reference 16 discusses further techniques to get the best performance from the ERA method. Similar ideas could be applied within the ERA/DC method.

ERA/DC Method

The implementation of this method depends on the structure of the R matrix that is the product of two block matrices. The simplest approach is to use $\xi = 0$ so that H becomes a row of block matrices and the $NO*NO$ matrix

$$R_k = \sum_{\ell=1}^{\eta+1} Y_{\ell+k} Y_\ell^T \tag{21}$$

defines the approximate correlations for a lag value of k. It can be shown that this matrix is the summation, over all of the input cases NI, of a matrix of output autocorrelations and crosscorrelations between outputs.

Because of the block layout of the U matrix, only the correlations for lags q to $q + (\alpha + \beta)r$ need to be calculated. The simplest layout occurs for a value of $r = 1$. However, it is possible to increment the lag values of the correlation blocks differently in the row and column directions, just as discussed earlier for the H matrix in the ERA method; any savings in computation would probably be of the same order.

Note that, unlike the ERA method, it is not actually necessary to store the H matrix but only the smaller U matrix.

The degree of initial model order overspecification, if any is required, is dictated by the smaller dimension of U, namely the number of rows $(\alpha + 1)NO$; it must be at least M, but a little overspecification in the presence of noise does seem to help.[11]

The greater dimension of U, the number of columns $(\beta + 1)NO$, is an indication of the number of correlations included in the fit. The choice of parameters must be such that

$$\eta + \alpha + \beta + q + 1 \le NP \tag{22}$$

where from Eq. (21) η is a measure of how many data points are used to calculate each correlation (typically one-half to two-thirds of NP).

One disadvantage of this formulation is that for a large number of outputs NO, the R blocks within the U matrix will be large and that, for a given matrix size, the number of correlation lags will be comparatively small. This limitation can be avoided by an alternative formulation.

Alternative ERA/DC Formulation

Instead of using Eq. (13) to define the correlation matrix, a different definition

$$\tilde{R}_k = H_o^T H_k = W^T V^T V A^k W = V_c A^k W \tag{23}$$

may be used and the process repeated as before using $\tilde{R}$ blocks in the U matrix. This alternative approach is discussed in the Appendix and yields a realization for A, $V_c A^q$, and W.

It can be shown, choosing that $\eta = 0$, that the $NI*NI$ matrix

$$\tilde{R} = \sum_{\ell=1}^{\zeta+1} Y_\ell^T Y_{\ell+k} \tag{24}$$

is actually the summation, over all of the outputs NO, of a matrix of output autocorrelations for each input and crosscorrelations between the same output for different inputs.

Since in general $NI \ll NO$, then this revised formulation will allow many more correlation lag values to be included in a given size U matrix. This would appear to have a similar effect to the fairly common practice of assuming reciprocity to reduce matrix sizes but without that assumption being made. It seems to allow the same information to be "packed" into a smaller matrix.

This formulation has not been proposed before and will be the subject of future investigation.

Noise Considerations

In practice it can be expected that some measurement noise will be present in the data. A detailed discussion on the effects of noise is beyond the scope of this paper, but some brief observations will be made for "white" measurement noise.

It is well known that the zero lag autocorrelation value of an output is corrupted by an additional term related to the mean square of the noise present. Fundamentally, biased parameter estimates are caused by this corrupted autocorrelation.[10] In the case of the ERA/DC method, the zero lag correlation matrix may be omitted by choosing the parameter $q \ge 1$ when $\zeta = 0$. Thus estimates will be unbiased for white noise.

The effect of noise on the ERA method may be seen by recognizing that it can in fact be shown[11] that the ERA method is a special case of the normal ERA/DC method with $q = \alpha = \beta = 0$ and $(\xi + 1)NO \ge M$. Thus, from Eq. (14), $U_o = R_o$, which means that bias will occur.

Computational Efficiency

The computation involved in the two methods may be considered in three stages: 1) setting up the block matrices H or U, 2) carrying out the singular value decomposition, and 3) obtaining state space realization, eigenvalue solution, and modal parameters.

For stage 1 there is little effort in the ERA method, but it can be shown that the ERA/DC method requires a number of multiplications equal to (number of elements in H matrix) * (sum of dimensions of U matrix) to calculate the required R matrices. Clearly a doubling of the number of data points in H, or of the dimensions of U, will only lead to about a doubling of the computation involved.

A guideline to the time taken by the SVD process is given in the NAG numerical library; for a $p*n$ matrix $(p \ge n)$, the time is proportional to $n^2(p + 4n)$ where n will represent the overspecified model order. Clearly if the H or U matrix is doubled in size, the computing time increases eightfold so that the stage 2 computation will increase much more rapidly with matrix size than that for stage 1. Also, if the ERA/DC method can yield results similar to the ERA method by using a block matrix of the order of half the size, then the computational effort in stage 2 will be less by a factor of eight. Depending on the size of the problem, this may outweigh the penalty of more stage 1 computation. Even for the $NO = NI$ case, the ERA/DC method requires a much smaller Hankel matrix than the ERA technique and therefore requires much less computation for the SVD stage of the calculation to obtain the same quality of results.

Finally, if it is assumed that the singular values retained after truncation are the same for each method (more or less equal to the true model order M), then it is expected that the computation involved in stage 3 will be approximately proportional to M^3 and about the same for both methods.

The relative computational performance of the methods will depend on the problem size and the degree of overspecification required. Some timings will be presented later.

Simulation Model

To be able to apply the methods to data with close modes, data were generated using the simple planar Space Station type model shown in Fig. 1. For convenience a beam finite element representation was used to obtain modal parameters for data generation. Solar arrays were idealized as T-beams to allow for their fundamental bending and torsional motion. Modules that might be attached to the main truss structure were represented by small grids of heavy and stiff beams. Only the first eight flexible modes that involved out-of-plane Z motion were included in the simulation and 2% critical damping added arbitrarily to each mode; the natural frequencies were 0.2825, 0.2834, 0.2869, 0.3131, 0.4329, 0.5513, 0.6827, and 0.7732 Hz. Note that the six rigid-body modes were excluded from the data; if acceleration measurements could be used, then rigid-body acceleration components would be zero during the free decay following an impulsive type input.

Only two excitation patterns were used, one with equal positive impulses at the four corner positions A–D shown in Fig. 1 and one with positive impulses at A and C but negative impulses at B and D. Thus two sets of responses ($NI = 2$, $NO = 8$) were generated at time intervals of $\Delta t = 0.4$ s (approximately 3.5 points per cycle at the highest frequency of interest). The lowest frequency component will decay to around one-fourth of its amplitude in 40 s so that an NP value of 100–125 is reasonable.

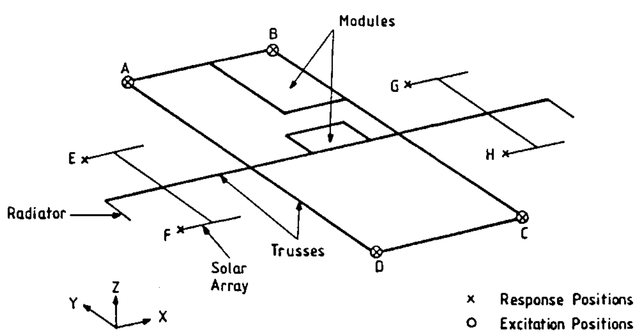

Fig. 1 Diagram of a simulated Space Station model.

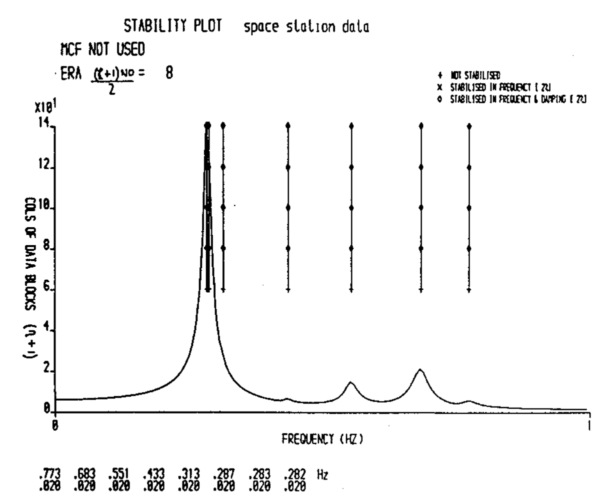

Fig. 2 ERA results for noise-free data, varying the number of columns in the H matrix.

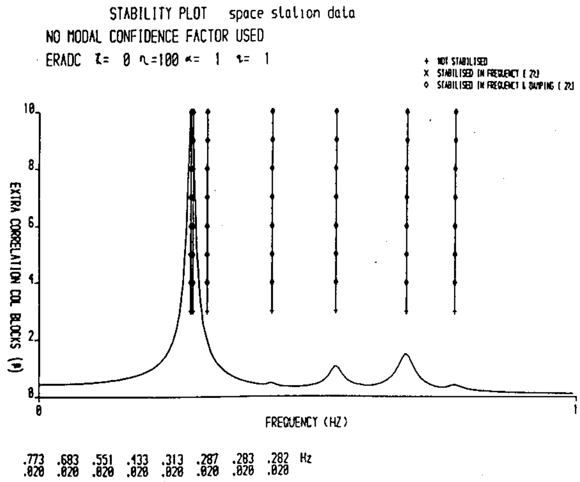

Fig. 3 ERA/DC results for noise-free data, varying the number of columns in the U matrix.

Results

For convenience, results will be presented in the form of stability plots[12] that allow the convergence of estimates to be examined when a particular parameter is varied. Such plots help in choosing suitable parameter values and also in detecting spurious modes. However, for the purposes of this paper, the singular values will be truncated to the known model order of 16 (2*8 modes) so that no spurious results are generated. The plot symbols indicate which modal parameters have converged within a given tolerance compared with the previous estimate. The vertical lines on each plot show which points are used to obtain the average frequency and damping ratio values presented below the plot. A composite power spectrum obtained from the transform of all of the output responses is also superimposed.

When applied to noise-free response data, accurate parameter estimates were provided by both methods for all eight modes, using one or two inputs and no model overspecification; the single input worked because none of the roots were exactly coincident. Some sample plots for the noise-free data using two inputs are presented in Figs. 2 and 3 for the ERA and ERA/DC methods, respectively; in each case there was no overspecification, and the parameter varied was related to the number of columns in the block matrices. Damping and frequency values were very accurate for all parameter values.

It should, however, be pointed out that the sensitivity of time domain methods to noise and rounding errors, for systems with very close, lightly damped modes, depends critically on the sample interval Δt. In fact, as large a value of Δt (as few points per cycle) as possible should be chosen to minimize sensitivity problems and reduce the severity of bias[17] because time domain methods are essentially exploiting the change in response across one sample interval. The sensitivity problem can therefore restrict the frequency bandwidth over which modes can be identified simultaneously; in the polyreference method, for example, narrow frequency bands are analyzed by transforming a band-limited frequency response function with consequent leakage errors. If the sample interval Δt is too small, then rounding errors can be a problem for the ERA/DC method because the SVD of a matrix product is carried out. However, larger bias errors are obtained for the ERA method. An alternative discussion of sensitivity problems for high sample rates is given in Ref. 18.

In the authors' experience, there have been no problems due to numerical ill-conditioning with the ERA/DC method. Although the formulation of the Hankel matrix involves correlations that would appear to infer a doubling of the condition number, as the user is able to define which correlations are used, and much less model order overspecification is used than least squares based approach, numerical problems have not occurred.

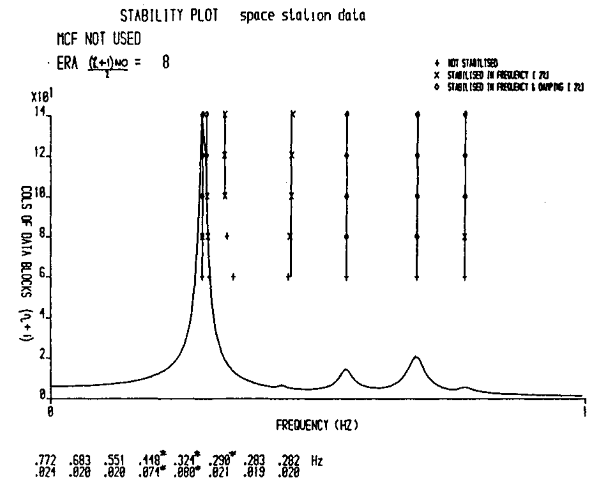

Fig. 4 ERA results for noisy data, varying the number of columns in the H matrix.

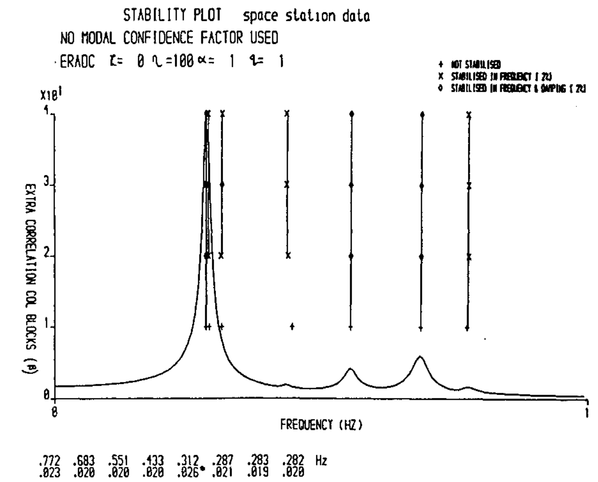

Fig. 5 ERA/DC results for noisy data, varying the number of columns in the U matrix.

The response data were then corrupted by truncating each sample value to around three to four significant figures and so adding quantization type noise, sufficient to introduce some bias into the estimates.

Results from using only one input were quite reasonable but are not presented here because one of the three close modes was very poorly excited and yielded inaccurate estimates that tended to confuse the interpretation of the other results.

However, results for the two input case were much better since the close modes were adequately excited. Some sample results are presented in Figs. 4 and 6 for the ERA method and Figs. 5 and 7 for the ERA/DC method; Figs. 4 and 5 show the effect of using more *columns* in the block matrices (i.e., more data points or correlation values) for no overspecification, whereas Figs. 6 and 7 show the effect of overspecifying the model by changing the number of *rows* in the block matrices.

A comparison of Figs. 4 and 5 shows that there is some improvement in the results as the number of columns in the block matrices is increased and that the bias that appears on some of the ERA frequency and damping results (shown with an asterisk) is largely absent for the ERA/DC method, as might be expected. The ERA bias is far less severe than that seen in Ref. 11 for a simpler single-input/single-output system, presumably because of differences in the signal-noise ratio, the type of noise used, and damping levels.

When the model is overspecified, as seen in Figs. 6 and 7, then the ERA results improve and the bias is overcome by using around double the model size (overspecification of 2*). The ERA/DC results are largely unchanged by overspecification although the slight bias seen in Fig. 5 is overcome. Note that some statistical variation in results is expected due to the presence of noise.

To gain some appreciation for the relative computation involved in the two methods, some sample timings were obtained on a VAX 11/750 computer with a Floating Point Accelerator (0.7 Mips). The timings refer to the CPU usage and are presented in Figs. 8 and 9; the symbols 1, 2, and 3 refer to the stage of computation.

In Fig. 8, the effect of increasing the number of columns in the block data matrices is seen, with no overspecification used. Here the time required to run the ERA/DC method is significantly slower because of the effort involved in calculating correlations at stage 1. However, it should be pointed out that these timings are for a single calculation and that there would be considerable savings for the ERA/DC method when several analyses were carried out using different parameter settings, as is the case in practice.

In Fig. 9, the effect of the model overspecification factor on the timings is shown. Because overspecification increases the number of rows in the block matrices on which the SVD is carried out, the stage 2 time increases rapidly, far more so than the time for the other two stages. The rate of increase of the stage 2 timing is, however, far less than would be expected from the guidelines given earlier; this shortfall is perhaps due to a fixed overhead time, relatively independent of the problem size.

Clearly the relative timing of the two methods depends on the degree of overspecification required. For this example, the

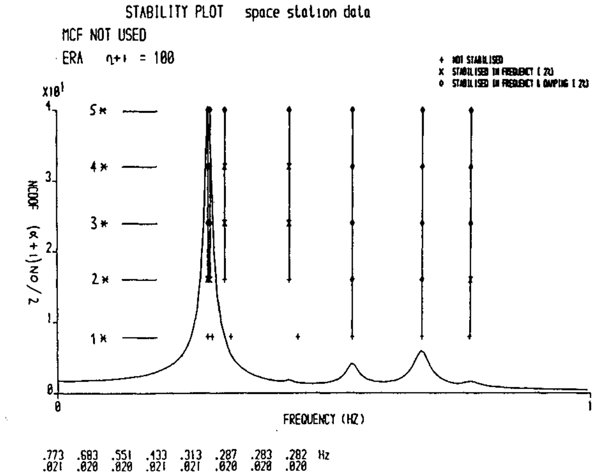

Fig. 6 ERA results for noisy data, varying the degree of model overspecification.

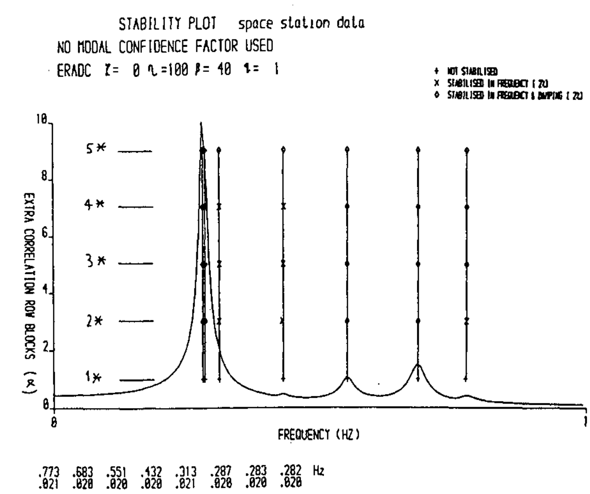

Fig. 7 ERA/DC results for noisy data, varying the degree of model overspecification.

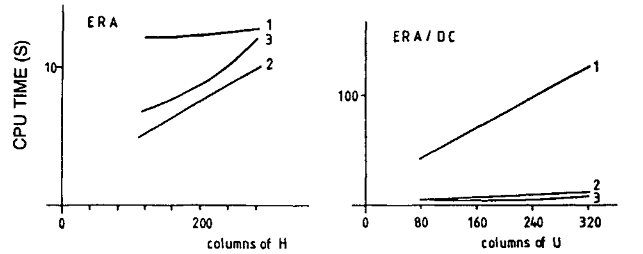

Fig. 8 Timings for a single analysis using no model overspecification.

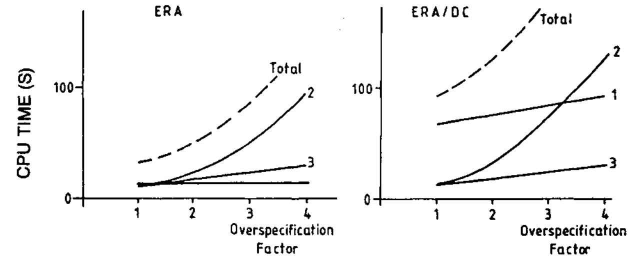

Fig. 9 Timings for a single analysis using an overspecified model order.

timings are almost the same for a single analysis if a 3* overspecification is used for the ERA method and none is used for the ERA/DC method. For multiple parameter cases, the timings would begin to favor the ERA/DC method provided that the correlations produced in stage 1 could be reused.

The timings shown here do not appear to be consistent with the comment in Ref. 11 that the ERA/DC method was around three to five times faster than the ERA method for multiple parameter run on single-input/single-output data. However, it would appear that the increase in the ERA/DC computation time seen in this paper is due to the use of MIMO data, such that the stage 1 computation increases by a factor of $NI*NO$ for an equivalent size U matrix and number of data points.

The revised formulation presented earlier was aimed at using only $NI*NI$ size blocks in the U matrix. If the same size U matrix were used, the computation would be similar, but the revised formulation allows the same number of correlation lag values to be used with a smaller U matrix so significant savings could result. Also in this formulation, it would be straightforward to follow the NASA practice of using two types of data block in the H matrix before the calculation of correlations, thus saving computation. This idea requires further consideration.

Conclusions

Both the ERA and ERA/DC methods are suitable for identification of state space MIMO models of spacecraft structures. Such structures will prove extremely difficult to identify due to the multitude of close modes, and future work into the design of suitable excitation inputs is required. The ERA/DC method caters to the bias problem without requiring the often considerable degree of initial model order overspecification of the ERA method. The computational performance of the two methods will be very similar in practice where multiple parameter settings are used. A revised formulation of the ERA/DC method has been presented and may be more efficient.

Appendix: Alternative ERA/DC Formulation

The block data correlation matrix is defined differently than that used in Eq. (13) such that

$$\tilde{R}_k = H_o^T H_k = W^T V^T V A^k W = V_c A^k W \tag{A1}$$

where $\tilde{R}_k$ is a square matrix of size γ where $\gamma = (\eta + 1)NI$.

The block correlation Hankel matrix U_q is now constructed such that

$$U_q = \begin{bmatrix} \tilde{R}_q & \tilde{R}_{q+r} & \cdots\cdots & \tilde{R}_{q+\beta r} \\ \tilde{R}_{q+r} & \tilde{R}_{q+2r} & \cdots\cdots & \tilde{R}_{q+(\beta+1)r} \\ \vdots & \vdots & & \vdots \\ \tilde{R}_{q+\alpha r} & \tilde{R}_{q+(\alpha+1)r} & \cdots\cdots & \tilde{R}_{q+(\alpha+\beta)r} \end{bmatrix} \tag{A2}$$

where α and β determine the size of the $(\alpha + 1)\ \gamma^*\ (\beta + 1)\gamma$ block correlation Hankel matrix. The q parameter is set so that the low lag autocorrelations can be omitted, and the r parameter enables the Hankel matrix to be set up without any overlap between any of the blocks.

The SVD of the Hankel matrix is

$$U_q = P_u D_u Q_u^T \tag{A3}$$

with pseudo-inverse

$$U^{\#} = Q_u D_u^{-1} P_u^T \tag{A4}$$

so that the expression

$$\tilde{R}_{q+j} = E^T U_{q+j} E \tag{A5}$$

can be expanded to give

$$\tilde{R}_{q+j} = E^T P_u D_u^{1/2} [D_u^{-1/2} P_u^T U_{q+1} Q_u D_u^{-1/2}]^j D_u^{1/2} Q_u^T E \tag{A6}$$

Equation (A6) can be compared with Eq. (A1) for $k = q + j$ such that

$$\tilde{R}_{q+j} = V_c A^{q+j} W = V_c A^q A^j W \tag{A7}$$

so

$$A = D_u^{-1/2} P_u^T U_{q+1} Q_u D_u^{-1/2} \tag{A8}$$

It can also be seen that

$$W = D_u^{1/2} Q_u^T E$$

and the first NI columns of the W matrix give the modal participation factors. Knowing that $H_o = VW$, then

$$V = H_o W^T (W W^T)^{-1} \tag{A9}$$

and the first NO rows of the V matrix give the mode shapes.

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